

XII Determinants, matrices, differentiation ,integration, inv trig.

1. Prove that i. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$. ii. $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3+b^3)^2$.

2i. Find A^{-1} if $A = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$ and hence solve $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

ii. $\begin{vmatrix} a & -3 & a+c \\ 3d & 0 & -2 \\ -5 & b & 0 \end{vmatrix}$ be a skew matrix, find a, b, c, d.

3i. Prove that $\frac{1}{2} \tan^{-1} x = \cos^{-1} \frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}$ ii. Show that $\cot^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \pi/2$.

iii. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

4.i. Find dy/dx if $x^y + y^x = 5$. ii. If $y = (\cos^{-1} x)^2$ show that $(1-x^2) y_2 - xy_1 = 2$. iii. If $x = a \sin^3 t$, $y = a \cos^3 t$, find y_2 .

5. . Integrate: i. $\int \frac{6x+5}{\sqrt{6+x-2x^2}} dx$. ii $\int \frac{dx}{e^{2x}+1}$. iii. $\int \frac{1}{3\cos x + 2} dx$.